M.A./ M. Sc. Mathematics Fourth Semester Updated Syllabus
Deen Dayal Upadhyaya Gorakhpur University
Paper – I
Lebesgue Integration

Lebesgue Integration of a simple measurable function on R and its properties. Lebesgue Integration of a bounded measurable function on a set E with finite Lebesgue measure, i.e. λ (E) $<\infty$, and its properties. *Bounded convergence theorem*, Lebesgue integration and Riemann integration.

(1.5 questions)

Integration on a measure space. Integration of a non-negative measurable function on a measure space. Lebesgue integral of general measurable function and its properties. Space of Lebesgue integrable functions.

Integral as a measure. Lebesgue's *monotone convergence theorem*. Integral of sum of two non-negative measurable functions. Integral as a countably additive set function. Integral of a non-negative function on a set of small measure. Integral of sum of two measurable functions. Term by term integration of a series of non-negative measurable function. *Fatou's lemma*. Lebesgue's *dominated convergence theorem*.

(2.5 questions)

Product Measure. Measurable rectangles. Semi algebra of measurable rectangles, Product measure and product measure space. Measurability of a section of a set in $R_{\sigma\delta}$. Measurability of a section of measurable set with finite product measure. *Fubini's theorem*.

(1.5 questions)

L_p Spaces: L_p (X, M, μ) and L_p(X, M, μ) spaces as vector spaces. Norm on L_p (X, M, μ) spaces. *Holder's and Minkowski's inequalities*. L_p (X, M, μ) as a Banach space. Bounded linear functionals on L_p (X, M, μ).

Essentially bounded functions and their properties, L_∞ spaces and its completeness.

(1.5 questions)

Differentiations: *Dini's four derivatives*. Differentiation of monotonic functions, Integral of the derivative of monotonic increasing functions. Indefinite integral as a continuous function of bounded variation. Indefinite integral as an absolutely continuous function. Derivative of an integral. Fundamental theorem of the Integral Calculus for the Lebesgue integration.

(1.5 questions)

Books recommended:

1. Walter Rudin: Principles of Mathematical Analysis (3rd edition), McGraw-Hill, Kogakusha, 1976 International Student edition.
2. H. L. Royden : Real Analysis, Macmillan Pub. Co. Inc. New York, 4th Edition, 1993.
3. Richard Johnson Baugh; Foundation of Mathematical Analysis.

Reference books :

1. G. de Barra : Measure theory and Integration, Wiley Eastern Limited, 1981.
2. E. Hewitt & K. Stromberg : Real and Abstract Analysis, Springer – Verlag, New York, 1969.

M.A./ M. Sc. Forth Semester

Mathematics
Paper – II
Hilbert Spaces

Inner product spaces, their basic properties and examples, Schwartz inequality. Norm induced by inner product, Continuity of inner product, Hilbert spaces and their examples. Parallelogram equality, polarization identity. Characterization of inner product in terms of norm. Separable Hilbert spaces and their examples

(2.5 questions)

Orthogonal vectors. Orthogonal complement. Projection theorem. Projection operators. Orthogonal sets and their advantage over its linearly independent sets. Complete orthonormal sets. *Bessel's generalized inequality. Parseval's Relation. Grahm-Schmidt orthogonalization process.* Fourier series representation

(2.5 questions)

Bounded linear functionals on Hilbert spaces. *Riesz-Frechet representation theorem*. Dual spaces. Inner product structure of dual spaces. Reflexivity of Hilbert spaces.

Hilbert adjoint operators. Shift operators. Special cases of Hilbert adjoint operoters self adjoint operators, positive operator, normal operators, unitary operators. Orthogonal projection operators.

(3 questions)

Book recommended :

P.K. Jain, O.P. Ahuja and K. Ahmad: Functional Analysis, New Age International (P) Ltd. and Wiley Eastern Ltd., New Delhi, 1997.

Reference books :

1. B. Choudhary.& S. Nanda: Functional Analysis with Applications, Wiley Eastern Ltd., 1989.
2. I.J Maddox: Functional Analysis, Cambridge University Press (1970).
3. G.F. Simmons: Introduction to Topology and Modern Analysis, McGraw-Hill Book Company, New York, 1963.
4. K. Chandrashekhara Rao. Functional Analysis, Narosa Publishing House, New Delhi

M.A./ M. Sc. Fourth Semester
Mathematics
Paper – III
Discrete Mathematics

Semigroups & Monoids: Definition and examples of Semigroups and Monoids. *Homomorphism* of Semigroups and Monoids. *Congruence relation* and Quotient Semigroups. Sub-semigroup and Sub-monoids. *Direct products*. Basic homomorphism theorem.

Lattices: Lattices as partially ordered sets. Their properties. Lattices as Algebraic Systems. Sub-lattices. Direct products and Homomorphisms. Some Special Lattices e.g., Complete, Complemented and Distributive Lattices.

(4 questions)

Boolean Algebras: Boolean Algebras as Lattices, Various Boolean Identities. The Switching Algebra example. Sub-algebras. *Direct Products* and Homomorphisms. *Join-irreducible elements*: *Atoms* and *minterms/maxterms*. Boolean Forms and their Equivalence.

Graph Theory: Definition of Graphs, Paths, Circuits, Cycles & Subgraphs. Induced Subgraphs. Degree of a vertex. Connectivity. Planar graphs and their properties. Trees. Euler's Formula for connected planar graphs.

(4 questions)

Books recommended :

1. C.L. Liu: Elements of Discrete Mathematics (Second Edition), McGraw Hill, International Edition, Computer Science Series, 1986.
2. J.P. Tremblay & R. Manohar: Discrete Mathematical Structures with Applications to Computer Science, McGraw-Hill Book Co., 1997.
3. N. Dew. Graph Theory with Application to Engineering and Computer Sciences, Prentice Hall of India.

M.A./ M. Sc. Fourth Semester
Mathematics
Paper – IV
Analytical Dynamics

Classification of dynamical systems. Generalized coordinates. Holonomic and non-holonomic systems, Kinetic energy. Generalized components of momentum. Generalized Components of the effective and applied forces. Lagrange's equations. Examples, Energy equation from Lagrange's equation. Reciprocal relations. Lagrange's equation for impulsive motion. Ignoration of coordinates. The *Routhian function*. Euler's equation from Lagrange's equation.

(4 questions)

Hamilton's equations of motion. Application of Hamiltonian methods. Natural motions. The space of events. Action. Hamilton's principle. Principle of least action. Hamilton-Jocobi equation. Hamilton characteristic function. Generating function. Canonical transformations. Phase space. Bilinear invariants. Poisson brackets, Lagrange brackets. Invariance of Lagrange brackets and Poisson brackets under canonical transformations. Small oscillations. Langrange's equation of small oscillations. Lagrange's determinants. Normal modes and normal coordinates and their stationary properties.

(4 questions)

Books recommended :

1. S.L. Loney: An Elementary Treatise on the Dynamics of a Particle and a Rigid Body, Macmillan India Ltd., 1982.
2. A.S. Ramsey: Dynamics part-II, The English Language Book Society and Cambridge University Press, 1972.
3.J.L. Synge and B.A. Griffith: Principles of Mechanics, McGraw Hill International Book Company, 1982

M.A./ M. Sc. Fourth Semester
Mathematics
Paper – V (a)
Finsler Geometry

Finsler metric function. Its properties. Tangent space. Indicatrix. Metric tensor and C-tensor Homogeneity properties of g_{ij} and C_{ijk}. Dual tangent space. Geodesics. δ-differentiation. Partial δ-differentiation. Properties of partial δ-differentiation.

(3 questions)

Fundamental postulates of Cartan. Cartan's covariant derivatives and their properties. Geometry of paths. Berwald's covariant derivative and its properties.

(2.5 questions)

Commutation formula resulting from partial δ-differentiation. Other commutation formulae. Three curvature tensors of Cartan. Identities satisfied by curvature tensors including Bianchi identities.

(2.5 questions)

Books recommended :

1. H. Rund: The Differential Geometry of Finsler Spaces, Springer-Verlag, 1959.
2. M. Matsumoto: Foundations of Finsler Geometry and special Finsler spaces, Kaiseisha Press, Saikawa, Otsu, 520 Japan, 1986.

M.A./ M. Sc. Fourth Semester
Mathematics
Paper – V (b)
Fourier Series

Convergence problems Dirichlet's integral. Riemann-Lebesgue theorem. Convergence tests. Dini's test. Jordan's test. Dela Valle's Poussin's test. Relations between the tests of Dini, Jorden and Dela Valle's Poussin. A continuous function with a divergent Fourier series.

(4 questions)

Functions of the class L^2: Bssel's inequality. Parseval's theorem for continuous functions. Riesz-Fischer theorem. Properties of Fourier coefficients. Uniqueness of trigonometrical series (Treatment of the above to be followed from Titchmarsh's book). Summability of Fourier series. Summability of S[f] and $\hat{S}$[f] by the method of first arithmetic mean. Abel summability of S [f] and $\hat{S}$ [f]

(4 questions)

Books recommended :

1. G.H. Hardy: Divergent series, Oxford 1949.
2. E.C. Titchmarsh: Theory of functions (relevant portion of chapter XIII).
3. A. Zygmud: Trigonometric Series Vol. 1, Cambridge 1959.
4. I.J. Maddox: Elements of Functional Analysis, Cambridge University Press, 1970.

M.A./ M. Sc. Fourth Semester

Mathematics

Paper – V (c)

Structures on a Differentiable Manifold –II

Almost Contact Manifolds: Definition. Eigen values of F. Intergrability conditions of π_m, $\pi\square_m$ and π_1. Lie derivative. Normal contact structure. Affinily almost cosymplectic manifold. Almost contact 3-structure.

(2.5 - 3 questions)

Almost Grayan Manifolds: Introduction. D-conformal transformation. Particular affine connections. *Almost Sasakian manifold*. Quasi-Sasakian manifold. Almost contact Riemannian 3-structure. Almost co-quaternion Riemannian structure. (2.5 -3 questions)

Sasakian Manifolds: K-contact Riemannian manifold and its properties, Sasakian manifold and its properties. Properties of projective, conformal, concircular and con- harmonic curvatures in Sasakian manifold. Cosymplectic structure. (2.5 - 3 questions)

Books recommended :

1. R.S. Mishra: Structures on differentiable manifold and their applications, Chandrama Prakashan, Allahabad, 1984.

2. K. Yano and M. Kon: Structure of Manifolds, World Scientific Publishing Co. Pvt. Ltd., 1984.

M.A./ M. Sc. Fourth Semester

Mathematics

Paper – V (d)

Wavelet Theory-II

Multiresolution Analysis for Wavelets: The unitary folding operators and the smooth projections. Multiresolution analysis and construction of wavelets. Construction of compactly supported wavelets and estimates for its smoothness. Band limited wavelets. Characterization of Lemarie-Meyer wavelets and some other characterizations.

(4 questions)

Spline Wavelets: Franklin wavelets and Spline wavelets on the real line. Orthonormal bases of piece-wise linear continuous functions for L^2 (T). Orthonormal bases of periodic splines. Periodization of wavelets defined on the real line. The basic equations and some of its applications. Characterization of MRA wavelets, low-pass filters and scaling functions.

(4 questions)

Book recommended :

Eugenio Hernandez and Guido Veiss: A First Course of Wavelets, CRC Press, New York, 1996.

Reference books:

1. C.K. Chui: An Introduction to Wavelets, Academic Press, 1992.

2. I. Daubechies: Ten Lectures on Wavelets, CB5-NSF Regional Conference in Applied Mathematics, 61, SIAM, 1992.

3.Y. Mayer: Wavelets, algorithms and applications (translated by R.D. Rayan, SIAM, 1993).

4. M.V. Wckerhauser: Aadopted wavelet analysis from theory of software, Wellesley, MA, A,K. Peters, 1994.

M.A./ M. Sc. Fourth Semester
Mathematics
Paper – V (e)
Special Functions-II

Bassel's differential equation and its series solution. Recurrence formula for $J_\nu(z)$. Schlaffi's contour integral for $J_\nu(z)$, Bessel function for integral order. Generating function for $J_\nu(z)$. Solution of Bessel's equation by complex integral. Hankel's functions.
Connection between Bessel and Hankel functions. The complete solution of Bessel's equation. Neumann''s polynomials and Neumann's expansion theorem. (4 questions)

The elliptic functions of Weierstrass: Periodic functions. Lower bound of the period of an analytic function. Definition of an elliptic function. The irreducible poles and zeros of an elliptic function and properties. Weierstrass's sigma functions. Zeta function. Weierstrass's elliptic functions and their properties.

(4 questions)

Book recommended :

1. E.T. Copson: Theory of Functions of a Complex Variable (Chapter IX and XIV).

M.A./ M. Sc. Fourth Semester

Mathematics

Paper – VI (a)

Bio-Mathematics

Introduction, Definition and Scope of Bio-Mathematics. Role of Mathematics in Biosciences.

Basic concepts of Fluid Dynamics. Basic concepts about blood. Cardiovascular system and blood flows. Blood flow through artery with mild stenosis. Two-layered flow in a tube with mild stenosis.

Peristaltic flow in tubes and channels.

(3 questions)

Gas exchange and air flow in lungs. Consumption and transport of Oxygen. Weibel's model for flows in lung airways. Comparison between flows of blood and flows in lung airways.

Diffusion. Fick's laws of diffusion. Diffusion equation. Modification of the diffusion equation. Diffusion in artificial kidney. Hemodialyser. Types of Hemodialyser.

(3 questions)

Books recommended :

1. J.N. Kapur: Mathematical Models in Biology and Medicine, Affiliated East-West Press Pvt. Ltd., New Delhi, 1985.

2. Y.C. Fung: Bio-Mechanics, Springer-Verlag New York Inc., 1990.

3. Stanley E. Charm and George S. Kurland: Blood Flow and Microcirculation, John Wiley & Sons, 1974.

4. S.A. Levin: Frontiers in Mathematical Biology, Springer-Verlag, 1994.

M.A./ M. Sc. Fourth Semester
Mathematics
Paper – VI (b)
Cosmology

Static Cosmological Models: Mach's principle. Einstein modified field equations with Cosmological term. Static cosmological models. Properties of Einstein model. Properties of De-sitter model. Difference between Einstein and De-sitter universe. Comparison of Einstein and De-sitter universes with actual universe.

(2 questions)

Non-Static Cosmological Models: Weyl's postulate. Cosmological principles. Hubble's law. Derivation of Robertson-Walker metric. Gemetrical feature of R-W metric. Redshift versus distance relation. Angular size versus redshift relation and source counts in Robertson-Walker space-time. Friedmann models. Particle horizon. Event horizon. Einstein's equation and dynamics of the universe.

(2 questions)

Origin & Evolution of Universe: Closed and open universes. Age of the universe. Origin and evolution of universe. Creation of matter. C-field theory. The Action principle. Cosmological equation. Explosive creation. Matter dominated era of universe. The large number hypothesis. Observable parameters of the steady state theory. Gravitational collapse. Gravitational collapse of homogeneous dust ball. Black holes (S trong gravitational fields).

(2 questions)

Books recommended :

1. Lowden F.C. : Relativity.
2. J. V. Narlikar : General Relativity & Cosmology.
3. S. R. Roy & Rajbali : Theory of Relativity.

M.A./ M. Sc. Fourth Semester
Mathematics
Paper – VI (c)
Magneto Fluid Dynamics – II

Bernoulli's equation in MHD. Kelvin's theorem in MHD. Ferraro's law of isorotation. Force-free magnetic fields. Pinch confinement of a plasma. Steady flow through a nozzle. Compatibility condition across a general discontinuity surface. Jump conditions across MHD shocks. Generalised Hugoniot condition. Compressible nature of MHD shocks. Stationary normal MHD shocks. Cylindrical shock waves generated by instantaneous energy release.

(3 questions)

Gasdynamic characteristic equations and characteristic curves. MHD characteristic equations. Fast and slow MHD waves. Transverse waves. Entropy waves. MHD simple waves. Contact surfaces and transverse simple waves. Fast and slow simple waves. Ordinary gasdynamic Characteristics in steady flow. Discontinuities in static case. Ordinary and Generalised Riemann invariants.MHD power generation.

(3 questions)

Books recommended:

1. Alan Jeffery, Magnetohydrodynamics, Oliver and Boyd Ltd., Edinburgh, 1966.
2. F. Chorlton, Text Book on Fluid Dynamics, C.B.S. Publishers, Delhi,1985.
3. S.I. Pai, Magnetohydrodynamics and Plasma Dynamics, Springer-Verlag, 1962.

M.A./ M. Sc. Fourth Semester
Mathematics
Paper – VI (d)
Viscous Flow

Fundamental equation of fluid dynamics of viscous compressible fluid. Equation of state. Equation of continuity. Equation of motion. Equations of energy.

(2 questions)

Exact solution of Navier-Stoke's equetion. Laminar flow between parallel walls (Couette and Poiseuille flow). Flow between concentric rotating cylinders-Couette flow. Hagen-Poiseuille flow in circular pipe. Convergent and divergent channels. Flow in the vicinity of stagnation point. Unsteady motioon of plate. One dimensional steady flow of vescous compressible fluid, Flow with uniform condition at + infinity. Shock wave thickness. Potential flow and Navier-Stoke's eqations. Vertically transport equation. Heat conduction equation. Linearized Navier-Stoke's equation. Limiting cases of every small and large Reynolds number.

(4 questions)

Book recommended :

1. S.I. Pai: Viscous Flow Theory : Laminar Flow.

M.A./ M. Sc. Fourth Semester
Mathematics
Paper – VI (e)
Mathematics of Insurance

Insurance Fundamentals: Insurance definition. Meaning of loss. Chances of loss, peril, hazard, and proximate cause in insurance. Costs and benefits of insurance to the society and branches of insurance. Insurable loss exposures-feature of loss that is ideal for insurance.

Life insurance Mathematics: Construction of mortality tables. Computation of premium of life insurance for a fixed duration and for the whole life.

(2 questions)

Determination of claims for General Insurance- using Poisson distribution and negative Binomial distribution- The Polya case. Determination of the amount of claims in general insurance-compound aggregate claim model and its properties and claims of reinsurance. Calculation of a compound claim density function. F-recursive and approximate formulae for F.

(2 questions)

The claim number process. The claim size process. Solvability of the Portfolio. Reinsurance and Ruin Problem. Premium and Ordering of Risks-Premium. Calculation Principles and Ordering Distributions. Ordering Distributions. Distribution of Aggregate Claim Amount-Individual and Collective Model. Compound Distributions.

(2 questions)

Books recommended :

1. Robert. J. Elliott and P. Ekkehard Kopp 'Mathematics of Financial Markests,' Springer-Verlag, New York Inc.
2.C.D. Daykin, T Pentikainen and M. Pesonen: Practical Risk Theory for Actuaries, Champman and Hall.
3. Tomasz Rolski, Hanspter Shmidli, Volker Schmidt and Jozef Teugels: Stochastic Processes for Insurance and Finance, John Wiley & Sons Limited.