M.A. / M. Sc. III Semester (Mathematics)

(Effective from session 2011-2012)

The M.A./M.Sc. Final (Mathematics) will consist of two semesters, called third and fourth semesters. Their examinations will be held in the months of December and April respectively. In each of these semester examinations there will be four compulsory papers and two optional papers—each one being selected from two separate groups of optional papers, marked Group 1 and Group 2. Each paper will be of three hours' duration and of 50 maximum marks, except where stated otherwise. There will be a viva-voce and project work examination of 50 marks in the fourth semester.

Format of the Question Paper.

There will be one compulsory question consisting of 5 parts of short answer type questions based on the whole course, out of which all parts will have to be answered. Besides, there will be 8 questions, ordinarily consisting of two sections (a) and (b), out of which 4 questions (2 from each) will have to be answered. Thus in all 5 questions will have to be attempted and 9 questions will have to be set. All questions will carry equal marks, except where stated otherwise.

Third Semester

Compulsory Papers

Paper I	:	Advanced Real Analysis
Paper II	:	Banach Spaces
Paper III	:	Advanced Complex Analysis
Paper IV	:	Dynamics of Rigid Bodies

Optional Papers

Group 1: Any one of the following will have to be opted.

Paper V (a)	:	Advanced Riemannian Geometry
Paper V (b)	:	Theory of Summability
Paper V (c)	:	Structures on a Differentiable Manifold-I
Paper V (d)	:	Wavelet Theory-I
Paper V (e)	:	Special Functions-I

Group 2: Any one of the following will have to be opted.

Paper VI (a)	:	Mathematical Modelling
Paper VI (b)	:	General Relativity
Paper VI (c)	:	Magneto Fluid Dynamics-I
Paper VI (d)	:	Theory of Elasticity
Paper VI (e)	:	Application of Mathematics in Finance

Viva –Voce and Project Work: **50 marks**

There will be a Viva-Voce and Project Work examination based on all the 12 papers of M.A./M.Sc. Final (Mathematics). Under the project, the candidate will present a dissertation in his/her own handwriting. The dissertation will consist of one theorem/article with proof or one problem with solution, relevant definitions with examples and/or counter-examples, wherever necessary, from each paper of Mathematics studied in third and fourth semesters. The dissertation will of 20 marks and the viva-voce will be of 30marks. For viva-voce examination and evaluation of project work there will be a board of examiners consisting of a coordinator, an external examiner and an internal examiner. The dissertation will be forwarded by the Head of Department at the university centre and by the Principal of the college at the college centre.

M.A./ M. Sc. Third Semester
Mathematics
Paper – I
Advanced Real Analysis

Sequences and series of functions of real numbers. Pointwise convergence and uniform convergence. Cauchy Criterion of uniform convergence. Weierstrass test for uniform convergence. Uniform convergence and continuity. Uniform convergence and integration. Uniform convergence and differentiation. Example of a function which is continuous everywhere on the real line but nowhere differentiable.

Functions of bounded variation and their properties. Absolutely continuous functions and their properties. Relation between absolute continuity and function of bounded variation.

Riemann-Stieltjes integration w.r.t. arbitrary integrator. Existence of Riemann-Stieltjes integrals. Integration by parts theorem. Properties of R-S integrable functions. Interchange of integrand and integrator functions. Uniform convergence and R-S integration. Evaluation of R-S integrals. R-S integrals and sequence of integrator functions. (3 questions)

Inadequacy of Riemann integration. Lebesgue's outer measure λ and its properties. Length of an interval and Lebesgue outer measure. Lebesgue measurable sets in R and σ-algebra of Lebesgue measurable sets M_λ in R Lebesgue measurability of open sets, closed sets and Borel sets. Labesgue measure on R. Example of a Non-Lebesgue measurable set. Cantor's set and its Labesgue measure.

General outer measure μ. Caratheodory's definition of μ-measurable sets. σ-algebra of μ-measurable sets M_{μ}. Definition of a measure. Measurable space and a measure space. Extension of a measure on an algebra to an outer measure.

(2.5 questions)

Definition of a measurable function. Equivalent conditions for measurable function. Sum and product of measurable functions. Composition of a measurable and a continuous function. Sequences of measurable functions. Measurability of supremum function, infimun function, limit superior function, limit inferior function and limit function. Simple measurable functions and their properties. A non-negative measurable function as the limit of a sequence of non-negative simple measurable functions. Concept of almost everywhere (a.e.). Lebesgue theorem. Measurability of Riemann integrable functions.

Convergence in Measure and its properties. F. Riesz theorem and Egorov theorem. Convergence almost everywhere, almost uniform convergence and their inter-relations (2.5 questions)

Books recommended :

1. Walter Rudin: Principles of Mathematical Analysis (3rd edition), McGraw-Hill, Kogakusha, 1976 International Student Edition.
2. H. L. Royden : Real Analysis, Macmillan Pub. Co. Inc. New York, 4th Edition, 1993.
3. Richard Johnson Baugh; Foundation of Mathematical Analysis.

Reference books :

1. G. de Barra : Measure theory and Integration, Wiley Eastern Limited, 1981.
2. E. Hewitt & K. Strumberg: Real and Abstract Analysis, Springer – Verlag, New York, 1969.
3. http://gauravtiwari.org/category/math/

M.A./ M. Sc. Third Semester

Mathematics

Paper – II

Banach Spaces

Normed linear spaces, Banach spaces, their examples including R^n, C^n, $l_p(n)$, $1 \le p < \infty$, c_0, c, l_p, $1 \le p < \infty$, P [a,b], C[a,b]. Joint continuity of addition and scalar multiplication. Summable sequences and completeness. Subsapaces, Quotient spaces of normed linear space and its completeness.

Continuous and bounded linear operators and their basic properties. Normed linear space of bounded linear operators and its completeness. Various forms of and operator norm.

(4 questions)

Isometric isomorphism, Topological isomorphism. Equivalent norms. Finite dimensional normed spaces and compactness. Riesz Theorem, Bounded linear functionals Dual spaces. Form of dual spaces $(R^n)^*$, $(C^n)^*$, c_0^*, l_1^*, l_p^*, $1 < p < \infty$.

Hahn- Banach theorem for real and complex normed linear spaces and its simple consequences. Open mapping theorem and its simple consequences. Product normed space. Closed graph theorem. Uniform boundedness. Banach-Steinhaus theorem. Embedding and Reflexivity

(4 questions)

Book recommended :

P.K. Jain, O.P. Ahuja and K. Ahmad: Functional Analysis, New Age International (P) Ltd. and Wiley Eastern Ltd., New Delhi, 1997.

Reference books :

1. B. Choudhary.& S. Nanda: Functional Analysis with Applications, Wiley Eastern Ltd., 1989.

2. I.J Maddox: Functional Analysis, Cambridge University Press (1970).

3. G.F. Simmons: Introduction to Topology and Modern Analysis, McGraw-Hill Book Company, New York, 1963.

4. K. Chandrashekhara Rao. Functional Analysis, Narosa Publishing House, New Delhi

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Mathematics

Paper – III

Advanced Complex Analysis

Analytic continuation. Uniqueness of analytic continuation. Power series method of analytic continuation. Branches of many-valued function. Singularities of an analytic function. Riemann surfaces. Gamma function. Zeta Function. Principle of reflection. Hadamard's multiplication theorem. Functions with natural boundaries.

(3 questions)

Maximum-modulus theorem. Schwarz's lemma. Vitali's convergence theorem. Hadamard's three-circles theorem. Mean values of $|f(z)|$. Borel-Cartheodory theorem. Pharagmen- Lindel of theorem.

(2 questions)

Conformal representation. Linear (bilinear) transformations involving circles and half-planes. Transformations $w=z^2$, $w=(z+1/z)/2$, $w = \log z$, $w = \tan^2 (z/2)$ Simple function and its properties. The "1/4 theorem".

Radius of convergence of the power series. Analyticity of sum of power series. Position of the singularities.

(3 questions)

Books recommended :

1. E.C. Titchmarsh: Theory of Functions, Oxford University Press, London.

2. Mark J. Ablowitz and A.S. Fokas: Complex Variables: Introduction and Applications, Cambridge University Press, South Asian Edition, 1998.

3. R.V. Churchill & J.W. Brown. Complex Variables and Applications, 5th Edition McGraw-Hill, New York, 1990.

4. Shanti Narayan: Theory of Functions of a Complex Variable, S. Chand & Co., New Delhi.

M.A./ M. Sc. Third Semester

Mathematics

Paper – IV

Dynamics of Rigid Bodies

Harmonic oscillators. Effect of disturbing force. Damped and forced oscillations. Motion of a rigid body in two dimensions under finite and impulsive forces. Kinetic energy and moment of momentum in two dimensions. Rolling spheres and cylinders. Conservation of energy and momentum.

(4 questions)

Motion of a billiard ball. Equations of motion and their applications in three dimensions. Motion of a system of particles. Moving axes. Equations of motion in most general form. Momentum of a rigid body. Euler's equations of motion. Moment of momentum about instantaneous axis. Kinetic energy of a rigid body. Motion relation to earth's surface.

(4 questions)

Books recommended :

1. S.L. Loney: An Elementary Treatise on the Dynamics of a Particle and of Rigid Bodies, Macmillan India Ltd., 1982.

2. A.S. Ramsey: Dynamics Part-II, The English Language Book Society and Cambridge University Press, 1972.

3.J.L. Synge and B.A. Griffith: Principles of Mechanics, McGraw Hill International Book Company, 1982.

M.A./ M. Sc. Third Semester

Mathematics

Paper – V (a)

Advanced Riemannian Geometry

Hypersurfaces : Unit normal. Generalised convariant differentiation. Gauss's formulae. Curvature of a curve in a hypersurface. Normal curvature. Mean curvature. Principal normal curvature. Lines of curvature. Conjugate and asymptotic directions. Tensor derivative of the unit normal. Gauss characteristic equation and Mainardi-Codazzi equations. Totally geodesic hypersurfaces.

(3 questions)

Subspaces: Unit normals. Gauss's formulae. Change from one set of normals to another. Curvature of a curve in subspace. Conjugate and asymptotic directions. Generalisation of Dupin's theorem. Derived vector of a unit normal. Lines of curvature for a given normal.

(1.5 question)

Lie derivative: Infinitesimal transformation. The notion of Lie derivative. Lie derivative of metric tensor and connection. Motion and affine motion in Riemannian spaces.

(1.5 question)

Hypersufaces in Euclidean space: Hyperplanes. Hyperspheres. Central quadric hypersurfaces. Reciprocal quadric hypersurfaces. Conjugate radii. Any hypersurface in Euclidean spaces. Riemannian curvature of a hypershpere. Geodesics in a space of positive constant curvature.

(2 questions)

Books recommended :

1. C.E. Weatherburn: An Introduction to Riemannian Geometry and the Tensor Calculus, Cambridge University Press, 1966.

2. K. Yano: The Theory of Lie Derivatives an its Applications, North Holland Publishing Company, Amsterdam,1957.

3 R. S. Mishra: A Course in Tensors with Applications to Riemannian Geometry, Pothishala (Pvt.) Ltd., 1965.

M.A./ M. Sc. Third Semester
Mathematics
Paper – V (b)
Theory of Summability

Special method of summation. Norlund means. Regularity and consistency of Norlund means. Inclusion. Equivalence. Arithmetic means. Holder's means. Simple theorems concerning Holder's means. Cesaro means. Means of non-integral orders.

Simple theorems concerning Cesaro summability. Equivalence theorem.Cesero an Abel summability (theorems 63, 64, 65 and 66 from Hardy's 'Divergent series').

(4 questions)

Matrix summability: Ordinary summability of sequences by infinite matrices (Treatment of the above to follow from Maddox's book).

Multiplication of series: Multiplication of (C,K) summable series.

(4 questions)

Books recommended :

1. G.H. Hardy: Divergent series, Oxford, 1949.
2. E.C. Titchmarsh: Theory of Functions (relevant portion of chapter XIII).
3. A. Zygmud:Trigonometric series Vol. 1, Cambridge, 1959 (relevant portion of chapter XIII).
4. I.J. Maddox: Elements of Functional Analysis, Cambridge University Press, 1970(relevant portion of chapter 7).

M.A./ M. Sc. Third Semester
Mathematics
Paper – V (c)
Structures on a Differentiable Manifold –I

Almost Complex Manifolds : Elementary notions, Nijenhuis tensor Eigen values of F, Integrability conditions, Contravariant and covariant analytic vectors, F-connection, Quaternion Structure (2.5 questions)
Almost Hermit Manifolds: Definition, Almost analytic vector fields. Curvature tensor. Linear connections. Almost quaternion metric structure. (1.5 question)
Kaehler Manifolds: Definition. Curvature tensor. Affine connection. Properties of projective, conformal, concircular and conharmonic curvature tensors. Contravariant almost analytic vector. Quaternion Kaehler manifold. (2.5 questions)
Nearly Kaehler Manifolds: Introduction, Curvature identities, Almost analytic vectors. (1.5 question)

Books recommended :
1. R.S. Mishra: Structure on differentiable manifold and their application, Chandrama Prakashan, Allahabad, 1984.
2. K. Yano and M. Kon: Structures of Manifolds, World Scientific Publishing Co. Pvt. Ltd., 1984.

M.A./ M. Sc. Third Semester
Mathematics
Paper – V (d)
Wavelet Theory-I

Wavelet Transform and its Basic Properties : Introduction. Continuous wavelet transform and examples. Basic properties of wavelet transform. Discrete wavelet transform. Orthonormal wavelets. (4 questions)
Different Ways of Constructing Wavelets: Orthonormal bases generated by a single function. Balaon-Low theorem. Smooth projections on L^2 (R). Local sine and cosine bases and the construction of some wavelets. (4 questions)

Book recommended :
Eugenio Hernandez and Guido Weiss: A First Course of Wavelets, CRC Press, New York, 1996.
Reference books:
1. C.K. Chui: An Introduction to Wavelets, Academic Press,1992.
2. I. Daubechies: Ten Lectures on Wavelets, CB5-NSF Regional Conference in Applied Mathematics, 61, SIAM 1992.
3. Y. Mayer: Wavelets, algorithms and applications (Translated by R.D. Rayan, SIAM, 1993).

4. M. V. Wikerhauser: Adopted wavelet analysis from theory of software, Wellesley, MA, A,K. Peters, 1994.

M.A./ M. Sc. Third Semester
Mathematics
Paper – V (e)
Special Functions-I

The Gamma Function: Analytical characters. Euler's limit formula. Duplication formula. Eulerian integral of first kind, Canonical product. Asymptotic expansion. Hankel contour integral.

Hypergeometric Functions: Solution of homogeneous linear differential equation of order two. Second order differential equation with three regular singularities. Hypergeometric equation and its properties. Confluent hypergeometric equation.

(5 questions)

Legendre functions: Complete solution of Legendre's differential equation. Integral representations and recurrence formulae for $P_n(z)$, $Q_n(z)$. Legendre polynomials of large degree. Neumann's expansion theorem. Associated Legendre's function.

(3 questions)

Book recommended :

1. E.T. Copson: Theory of Functions of a Complex Variable (Chapters IX an XIV).

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Mathematics

Paper – VI (a)

Mathematical Modelling

Mathematical Modelling: Need, techniques, classification and simple illustrations of mathematical modeling. Limitations of mathematical modelling.

Mathematical Modelling Through Ordinary Differential Equations of First Order: Linear and Non-linear Growth and Decay models. Compartment models. Mathematical modelling in Dynamics through ordinary differential equations of first order. Mathematical modelling of geometrical problems through ordinary differential equations of first order.

(3 questions)

Mathematical Modelling Through System of Ordinary Differential Equations of First Order: Mathematical modelling in Population Dynamics. Mathematical modelling of epidemics. Compartment models. Mathematical modelling in Economics.

Mathematical models in Medicine. Arm Race, Battles and International Trade in terms of system of ordinary differential equations, Mathematical modelling in Dynamics.

(2 questions)

Mathematical Modelling Through Ordinary Differential Equations of Second Order: Mathematical modelling of planetary motions. circular motion and motion of satellites, Mathematical modelling through linear differential equations of second order, Miscellaneous mathematical models through ordinary differential equations of second order.

Mathematical Modelling Through Partial Differential Equations: Situations giving rise to partial differential equation models. Mass-balance equations. Momentum-balance equations. Models for traffic flow on a highway.

(3 questions)

Book recommended:

1. J.N. Kapur: Mathematical Modelling, New Age International (P) Limited, New Delhi.

M.A./ M. Sc. Third Semester
Mathematics
Paper – VI (b)
General Relativity

Transformation of coordinates. Tensors. Algebra of tensors. Symmetric and skew-symmetric tensors. Contraction. Quotient law. Riemannian metric. Tensor density. Christoffel symbols. Covariant derivatives. Geodesics. Parallel transport. Riemannian Christoffel curvature tensor. Symmetric properties of R^{h}_{ijk},. Covariant curvature tensor. Ricci tensor. Bianchi identities.

(3 questions)

Uniform vector field. Flat space-time. Review of the special theory of relativity and the Newtonian. The law of gravitation. Principle of equivalence and general covariance. Geodesic principle. Newtonian approximation relativistic equations of motion, Emstein's field equations and its Newtonian approximation. Schwarzschild external solution and its isotopic form. (3 questions)

Planetary orbits and analogues of Kepler's laws in general relativity. Advance of perihelion of a planet. Bending of light rays in a gravitational field.

Gravitational reds hift of spectral lines. Rader Echo delay, Schwarzschild internal solution, Energy momcrtum tensor of a perfect fluid.

(2 questions)

Books recommended :

1. F.C. Lawdon : Relativity
2. J.V. Narlikar: General Relativity and Cosmology, The Macmillan Company of India Limited, 1978.
3. S.R Roy & Raj Bali : Theory of Relativity
4. A.S. Eddington: The Mathematical Theory of Relativity, Cambridge University Press, 1965.
5. P.G. Bergmann: Introduction to the Theory of Relativity
6. J.L. Synge: Relativity : the general theory, North Holland Publishing Company 1976.
7. C.E. Weatherburn: An Introduction to Riemannian Geometry and the Tensor Calculus, Cambridge University Press 1950.

M.A./ M. Sc. Third Semester
Mathematics
Paper – VI (c)
Magneto Fluid Dynamics – I

Maxwell equations. Electromagnetic field in a conductor. MHD approximations. Rate of flow of charge. Important MHD parameters. Diffusion of magnetic field. Frozen-in-fields. Integral of magnetic field equation. Analogy of magnetic field with vorticity. Alfven theorem. Lorentz force and its transformations. Magnetic energy. Poynting vector theorems. Basic equations of inviscid and viscous magnetohydrodynamics. Energy conservation law.

(4 questions)

Alfven waves. MHD waves in a compressible fluid. Equi-partition of energy of Alfven waves. MHD boundary conditions. Equations of incompressible MHD flow. Parallel steady flow. Steady parallel flow in a conservative field of force. One-dimensional steady viscous MHD flow. Hartmann flow. Couette flow.

(4 questions)

Books recommended :

1. Alan Jeffery, Magnetohydrodynamics, Oliver and Boyd Ltd., Edinburgh, 1966.
2. F. Chorlton, Text Book on Fluid Dynamics, C.B.S. Publishers, Delhi,1985.
3. S.I. Pai, Magnetohydrodynamics and Plasma Dynamics, Springer-Verlag,1962.

M.A./ M. Sc. Third Semester
Mathematics
Paper – VI (d)
Theory of Elasticity

Shear. Displacement. Strain and its varieties. Transformation of the components of strain. Relation connecting the dilatation, the rotation and the displacement. Resolution of any strain into dilatation and shearing strain.

Traction across a plane at a point. Surface tractions and body forces. Law of equilibrium of surface tractions on small volumes. Stress at point and its transformation. Stress quadric. Types of stress. Stress-equations of motion and of equilibrium.

(5 questions)

Strain energy function. Hooke's Law. Elastic constant. Determination of stress in a body. Strain energy function for isotropic solids of elasticity. Stress-strain relations.

(3 questions)

Book recommended :

1. I. Love: Mathematical Theory of Elasticity.

M.A./ M. Sc. Third Semester
Mathematics
Paper – VI (e)
Application of Mathematics in Finance

Financial Management: An overview. Nature and scope of financial management. Goals of financial management and main decision of financial management. Difference between risk, speculation and gambling.

Time Value of Money: Interest rate and discount rate. Present value and future value-discrete case as well as continuous compounding case. Annuities and its kinds.

(2.5 questions)

Meaning of returns: Return as Internal Rate of Return (IRR). Numerical methods like Newton-Rapshon method to calculate IRR. Measurement of returns under uncertainity situations.

Meaning of risk: Difference between risk and uncertainty. Types of risks. Measurement of risk. Calculation of security and Portfolio Risk and Return-Morkowitz Omdel. Sharpe's Single Index Model-Systematic risk and Unsystematic Risk. Taylor Series and Bond Valuation. Valuation. Calculation of Duration and Convexity of Bonds.

(3 questions)

Financial Derivative: Futures. Forwards. Swaps and Options. Call and Put Option. Call and Put Parity theorem. Pricing of contingent claims through Arbitrage and Arbitrage theorem.

Pricing by Arbitrage: A Single Period Option Pricing Model. Multi Period Pricing Model-Cox-Ross-Rubinstein Model. Bounds on Option Prices.

(2.5 questions)

Books recommended :

1. Aswath Damodaran: Corporate Finance-Theory and Practice, John Wiley & Sons, Inc.
2. John C. Hull: Options, Futures and Other Derivatives, Prentice-Hall of India Pvt. Limited.
3. Sheldom M. Ross: An Introduction to Mathematical Finance, Cambridge University Press.
4. Salih N. Neftci: An Introduction to Mathematics of Financial Derivatives, Academic Press Inc.
5. Robert J. Elliott and P. Ekkehard Kopp: Mathematics of Financial Markets, Springer-Verlag, New York Inc.