

## Second Semester

### Compulsory Papers

|           |   |                                |
|-----------|---|--------------------------------|
| Paper I   | : | Fields and Modules             |
| Paper II  | : | Topology-II                    |
| Paper III | : | Partial Differential Equations |
| Paper IV  | : | Differentiable Manifolds       |
| Paper V   | : | Fluid Dynamics                 |

### Optional Papers:

Any one of the following papers will have to be opted. Those who have offered Paper VI (a) in First Semester will offer Paper VI (a) in second semester, and similarly for other optional papers.

|              |   |  |
|--------------|---|--|
| Paper VI (a) | : | Spherical Astronomy-II                       |
| Paper VI (b) | : | Operations Research-II                       |
| Paper VI (c) | : | Fuzzy Logic                                  |
| Paper VI (d) | : | Numerical Methods                            |
|              |   | (i) Theory : 30 marks                        |
|              |   | (ii) Practical : 20 marks (2 hours duration) |
| Paper VI (e) | : | History of Mathematics-II                    |
| Paper VI (f) | : | Dynamics of Real Gases-II                    |

### Viva –Voce and Project Work:

**50 marks**

There will be a Viva-Voce and Project Work based on all the 12 papers of M.A./M.Sc. Previous (Mathematics). Under the project, the candidate will present a dissertation in his/her own handwriting. The dissertation will consist of one theorem/article with proof or one problem with solution, relevant definitions with examples and/or counter-examples, wherever necessary, from each paper of Mathematics studied in First and Second semesters. The dissertation will be of 20 marks and the viva-voce will be of 30 marks. For viva-voce examination and evaluation of project work there will be a board of examiners consisting of a coordinator, an external examiner and an internal examiner. The dissertation will be forwarded by the Head of Department at the university center and by the Principal of the college at the college center.

**M.A./ M. Sc. Second Semester**

**Mathematics**

**Paper I**

**Fields and Modules**

**Field theory** : Extension fields. Algebraic and transcendental extensions. Splitting field. Separable and inseparable extensions. Normal extension. Perfect fields. Finite fields. Automorphisms of extensions. Galois group. Fundamental theorem of Galois theory. Construction with ruler and compass. Solution of polynomial equations by radicals. Insolvability of the general equation of degree 5 by radicals.

(5 questions )

**Modules** : Cyclic modules. Simple modules . Semi-simple modules. Schuler's lemma. Free modules. Noetherian and artinian modules. Hilbert basis theorem.

( 3 questions )

**Books recommended :**

1. I.N. Herstein : Topics in Algebra, Wiley Eastern Ltd., New Delhi, 1975.
2. P.B. Bhattacharya, S.K. Jain and S.R. Nagpaul : Basic Abstract Algebra (Second Edition ), Cambridge University Press, Indian Edition , 1997.
3. Surjeet Singh and Qazi Zameeruddin: Modern Algebra, Vikas Publishing House. Pvt. Ltd., 2005.
4. T.Y. Lam : Lectures on Modules and Rings, GTM, Vol. 189, Springer Verlag, 1999.
5. A.R. Vasishtha & A.K. Vasishtha : Modern Algebra, Krishna Prakashan Media (P) Ltd., Meerut .

**M.A./ M. Sc. Second Semester**

**Mathematics**

**Paper –II**

**Topology –II**

Separation axioms –  $T_0$ ,  $T_1$ ,  $T_2$ , regular,  $T_3$ , normal and  $T_4$  spaces, their comparison and examples, hereditary and topological invariant characters. Urysohn's lemma and Tietze extension theorem.

Separated sets. Connectedness in terms of separated sets. Characterization of connected sets in terms of open sets and closed sets. Closure of a connected set. Union of connected sets. Connected sets in  $\mathbb{R}$ . Continuity of a function and connectedness. Components and partition of space.

(2.5 questions )

Inadequacy of sequential convergence. Directed sets, nets and subnets and their examples Convergence of a net, characterisation of open sets, closed sets, closure, cluster point and limit point of a set in terms of net convergence. Hausdorffness and continuity of a function in terms of nets.

Definition of filter and its examples. Neighborhood filter. Comparison of filters. Filter base and subbase. Convergence of a filter. Ultra filters. Continuous functions and filters.

Net based on filter and filter based on net.

(3 questions)

Quotient topology, quotient space, quotient map, quotient space  $X/R$ ,

Finite product space, projection mapping.

Tychonoff product topology in terms standard subbase and its characterizations in terms of projection maps, continuous functions, Product of  $T_0, T_1, T_2$ , spaces. Connectedness and compactness, first and second countability for product spaces.

(2.5 questions)

**Books recommended :**

1. George F. Simmons : Introduction to Topology and Modern Analysis, Mc Graw-Hill Book Company 1963.
2. J.L. Kelley : General Topology, Van Nostrand, Reinhold Co., New York 1955.
3. K.D. Joshi : Introduction to General Topology, Wiley Eastern Ltd., 1983.
4. James R Munkres : Topology, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
5. S. Willard : General Topology Addison-Wesley, Reading, 1970.

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**Mathematics**

**Paper III**  
**Partial Differential Equations**

**Partial Differential Equations of the First Order :** Origin of first order partial differential equations. Lagrange's solution of first order linear partial differential equation. Non-linear partial differential equations of the first order. Cauchy's method of characteristics, Charpit's method and Jacobi's method.

(4 questions )

**Partial Differential Equations of Second and Higher Orders :** Origin of second order partial differential equations. Higher order partial differential equations with constant coefficients. Equations with variable coefficients. Classification of second order partial differential equations. Canonical forms. Solution of non-linear second order partial differential equations by Monge's method. Method of separation of variables for solving Laplace, wave and diffusion equations.

( 4 questions )

**Books recommended :**

1. A.R. Forsyth : A Treatise on Differential Equations
2. I.N. Sneddon : Elements of Partial Differential Equations.

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**Mathematics**  
**Paper – IV**  
**Differential Geometry of Manifolds**

Definition and examples of differentiable manifold. Differentiable functions. Differentiable curves .Tangent space .Vector fields,.Lie bracket.

( 1 question )

Invariant view point of connections . Covariant differentiation . Torsion. Curvature . Parallelism . Difference tensor of two connections. Lie derivative.

( 3 questions )

Riemannian Manifold. Riemannian connection. Riemannian curvature tensor and Ricci tensor. Identities of Bianchi. Sectional curvature.

( 2 questions )

Exterior product of two vectors. Exterior algebra of order  $r$  .Exterior derivative .Cartans's structural equations.

( 1 question )

Submanifolds. Normals. Induced connection. Gauss formulae. Weingarten formulae. Lines of curvature. Mean curvature. Equations of Gauss and Codazzi.

( 1 question )

**Books recommended :**

1. B.B. Sinha : An Introduction to Modern Differential Geometry, Kalyani Publishers, New Delhi, 1982.
2. N.J. Hickls : Notes on Differential Geometry.
3. K. Yano and M. Kon: Structure of Manifolds, World Scientific Publishing Co. Pvt. Ltd.,1984.

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**Mathematics**

**Paper – V**  
**Fluid Dynamics**

Wave motion in a gas. Speed of Sound. Equation of motion of a gas. Subsonic, sonic and supersonic flows of a gas. Isentropic gas flows. Flow through nozzle. Shock formation. Elementary analysis of normal and oblique shock waves (4 questions )

Stress components in a real fluid. Relations between Cartesian components of stress. Rate of strain quadric. Principal stresses. Relations between stress and rate of strain. Coefficient of viscosity. Navier-Stokes equations of motion. Steady viscous flow between parallel planes and through tubes of uniform circular, elliptic and equilateral triangular cross-sections. Steady flow between concentric rotating cylinders. Diffusion of vorticity. Energy dissipation due to viscosity due to viscosity. Reynolds number. Prandtl's boundary layer. Karman's integral equation. (4 questions )

**Book recommended :**

1. F. Chorlton: Text Book of Fluid Dynamics, C.B.S. Publishers, Delhi, 1985

**M.A./ M. Sc. Second Semester**  
**Mathematics**  
**Paper – VI (a)**  
**Spherical Astronomy-II**

Planetary phenomena, geocentric motion of a planet, elongation, stationary points, phases, brightness of the planet.

(2 question )

Lunar and solar eclipses, Earth's shadow at moon's distance, ecliptic limits, greatest and least number of eclipses in a year.

(3 questions )

Determination of longitude and latitude, sextant, dip of the horizon, Mercator's projection, great circle on Mercator's chart, position circle.

(1.5 questions )

Proper motions and its effect in right ascension and declination, position angle, change in position angle due to star's motion and due to the motion of the pole, the motion of the sun, parallactic motion in right ascension and declination. Binaries.

(1.5 questions )

**Books recommended :**

1. Gorakh Prasad : Spherical Astronomy.
2. Ball : A Text book of Spherical Astronomy.

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**Mathematics**

## Paper – VI (b)

### Operations Research-II

**Input-Output Analysis :** Introduction, Meaning of input-output. Main features of analysis and assumptions. Leontief static model. Input-output table. Balance equation. Inter-industrial relation. Technological coefficient. Technology matrix. Problem based on changing demands.

(2 questions )

**Inventory control:** Introduction, Classification of inventory. Economic parameters associated with inventory problems. Deterministic models. Economic lot size model with uniform rate of demand. Sensitivity analysis of economic order quantity formula. Economic lot size with different rate of demand in different cycles. Economic lot size with finite rate of production. Limitation of EOQ formula. Deterministic model with shortage. Instantaneous production with back orders. Finite rate of replenishment of inventory. Fixed time model. Lost-sales shortages. Multi-item deterministic model with one linear constraint. Restriction on the number of stocked units, Restriction on the amount to be invested on inventory. Models with lead time.

(3 questions )

**Problems of replacement:** Introduction, Replacement models and their solutions. Concept of present value. Replacement of items whose efficiency deteriorates with time. Replacement of items whose maintenance cost increases with time and the value of money remains constant. Replacement of items when the value of money also changes. Criteria of present value for comparing replacement alternative. Staffing problem.

(2 questions )

**Dynamic Programming :** Formulation and solution of dynamic programming problem. Deterministic dynamic programming. Dynamic programming approach to problems of shortest route resource allocation and production.

(1 question )

#### Books recommended :

1. H.A. Taha: Operations Research – An Introduction, Macmillan Publishing Co., Inc., New York.
2. Kanti Swarup, P.K. Gupta, Man Mohan: Operations Research, Sultan Chand and Sons, New Delhi.
3. B.S. Goel, S.K. Mittal: Operations Research, Pragati Prakashan, Meerut.
4. P.K. Gupta, D.S.Hira: Operatons Research – An Introduction, S. Chand & Company Ltd., New Delhi.
5. S.S. Rao: Optimization Theory and Applications, Wiley Eastern Ltd., New Delhi.
6. J.K. Sharma: Operations Research – Theory and Applications, Macmillan India Ltd.
7. S.D. Sharma: Operations Research, Kedar Nath Ram Nath & Company.

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**Mathematics**

**Paper – VI (c)**

**Fuzzy Logic**

**Fuzzy Logic:** An overview of classical logic, Multi-valued logics. Fuzzy propositions. Fuzzy quantifiers. Linguistic variables and hedges. Inference from conditional fuzzy propositions, the compositional rule of inference.

(2 questions )

**Approximate Reasoning:** An overview of fuzzy expert system. Fuzzy implications and their selection. Multiconditional approximate reasoning. The role of fuzzy relation equation.

(2 questions )

**An Introduction to Fuzzy Control:** Fuzzy controllers. Fuzzy rule base. Fuzzy inference engine. Fuzzification and the various de-fuzzification methods (the center of area, the center of maxima, and the mean of maxima methods).

(2 questions )

**Decision Making in Fuzzy Environment:** Individual decision making. Multiperson decision making. Multi-criteria decision making. Multistage decision making. Fuzzy ranking methods. Fuzzy linear programming.

(2 questions )

**Books recommended :**

1. H.J.Zimmermann: Fuzzy set theory and its applications, Allied Publishers Ltd., New Delhi, 1991.
2. G. J. Klir and B. Yuan: Fuzzy sets and fuzzy logic, Prentice-Hall of India, New Delhi, 1995.

## Numerical Methods (with Programming in C )

### Theory :

Max Marks: 30

#### Numerical Solution of Systems of Linear Equations :

Introduction. Gauss- Elimination Method. Gauss- Elimination Method with Pivoting. III-Conditioned Equations. Iterative Refinement of the Solution obtained by Gauss Elimination Method. Gauss-Jordan Method. Crout's Method. Jacobi's Iteration Method. Gauss-Seidel Iteration Method. Successive over Relaxation (SOR) Method. Development of programs in C. (4 questions)

#### Numerical Solution of Partial Differential Equations:

Introduction. Finite- Difference Approximations to Derivatives. Classification of Partial Differential Equations. Elliptic Partial Differential Equations. Laplace's Equation. Solution of Laplace's Equation by Jacobi's Method, Gauss- Seidel Method, Successive Over-Relaxation (SOR) Method. ADI Method. Poisson's Equation, Solution of Poisson's Equation. Parabolic Partial Differential Equations. Heat Equation, Solution of Heat Equation. Crank-Nicolson Method. Hyperbolic Partial Differential Equations. Solution of Hyperbolic Partial Differential Equations. Wave Equation and its Solution by Finite- Difference Method, Development of programs in C. (4 questions)

#### Books Recommended:

- (1) E. Balagurusamy: Programming in ANSI C, TMH, New Delhi.
- (2) Prahlad Tiwari, R.S. Chandel and A.K. Tripathi : Programming in C & Numerical Analysis, Ram Prasad & Sons, Agra.
- (3) S.S. Sastry : Introductory Methods of Numerical Analysis, PHI, New Delhi.

### Practical :

Max. Marks :20

#### Numerical Methods (with Programming in C )

- 1 To implement Newton-Raphson method.
- 2 To implement Newton's forward/backward interpolation formula.
- 3 To implement Lagrange's interpolation formula.
- 4 To implement Trapezoidal rule.
- 5 To implement Simpson's one third rule.
- 6 To implement Gauss- elimination method
- 7 To implement Gauss- Jordan method.
- 8 To implement Crout's method
- 9 To implement Jacobi's method
- 10 To implement Gauss-Seidel method
- 11 To implement SOR method

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Mathematics

**Paper VI(e)**  
**History of Mathematics-II**

**Mathematics during nineteenth and twentieth centuries:** Synthetic geometry. Analytic geometry. Algebra. Analysis. Theory of functions. Theory of numbers. Applied Mathematics.

(8 questions)

**Book Recommended:**

(1) F. Cajon: A History of Mathematics

**M.A./ M. Sc. Second Semester**  
**Mathematics**  
**Paper – VI (f)**  
**Dynamics of Real Gases-II**

Linearized flow behind a normal shock wave. Quasi-one-dimensional flow. Dispersed shock wave, Nozzle flow.

(2 questions )

Flow with radiative non-equilibrium: Greyogas approximation. One dimensional equations. Differential approximation. Acoustic equation. Plane acoustic waves. Small departure from free stream. Linearized and non-linear flow through a normal shock.

(6 questions )

**Book recommended :**

1. W. G. and C.H.: Introduction to Physical Gas Dynamics.