Statistical Mechanics

I have planned to bring my class notes on internet now. In this very first article, I'd like to put some light on Ensembles which are the most essential parts of Statistical Mechanics. After the introduction of Ensembles, we shall proceed to other important topics like µ and γ spaces, postulates of Statistical Mechanics, Liouville's theorem, partition function etc. and much more. As always, I have tried my best to keep the language simple and graspable.

Ensembles:

As a system is defined by the collection of a large number of particles, so the "ensembles" can be defined *is* collection of a number macroscopically identical but essentially independent systems. Here the term *macroscopically independent* means, as, each of the system constituting an ensemble satisfies the same macroscopic conditions, like *Volume, Energy, Pressure, Temperature and Total number of particles etc.* Here again, the term *essentially independent* means the system (in the ensemble) being mutually non-interacting to others, i.e., the systems differ in microscopic conditions like *parity, symmetry, quantum states etc.*

There are three types of ensembles:

i. Micro-Canonical Ensemble
ii. Canonical Ensemble
iii. Grand Canonical Ensemble

Micro-canonical Ensemble

It is the collection of a large number of essentially independent systems having the **same energy E, volume V** and **total number of particles N.**

The individual systems of a micro-canonical ensemble are separated by rigid impermeable and insulated walls, such that the values of **E, V & N** are not affected by the mutual pressure of other systems.

This ensemble is as shown in figure:

System 1; Energy E Volume V Number of Particles N.	**System 2; Energy E Volume V Number of Particles N.**	**System 3; Energy E Volume V Number of Particles N.**	**System 4; Energy E Volume V Number of Particles N.**	**System 5; Energy E Volume V Number of Particles N.**
System 6; Energy E Volume V Number of Particles N.	**System 7; Energy E Volume V Number of Particles N.**	**System 8; Energy E Volume V Number of Particles N.**	**System 9; Energy E Volume V Number of Particles N.**	**System 10; Energy E Volume V Number of Particles N.**
System 11; Energy E Volume V Number of Particles N.	**System 12; Energy E Volume V Number of Particles N.**	**System 13; Energy E Volume V Number of Particles N.**	**System 14; Energy E Volume V Number of Particles N.**	**System 15; Energy E Volume V Number of Particles N.**
System 16; Energy E Volume V Number of Particles N.	**System 17; Energy E Volume V Number of Particles N.**	**System 18; Energy E Volume V Number of Particles N.**	**System 19; Energy E Volume V Number of Particles N.**	**System 20; Energy E Volume V Number of Particles N.**

System 21; Energy E Volume V Number of Particles N.	**System 22; Energy E Volume V Number of Particles N.**	**System 23; Energy E Volume V Number of Particles N.**	**System 24; Energy E Volume V Number of Particles N.**	**System 25; Energy E Volume V Number of Particles N.**

Here all the borders are impermeable and insulated.

Canonical Ensemble

It's the collection of a large number of essentially independent systems having the same **temperature T, volume V** and **the number of particles N**.

The equality of temperature of all the systems can be achieved by bringing all the systems in thermal contact. Hence, in this ensemble the individual systems are separated by rigid impermeable but **conducting** walls, the outer walls of the ensemble are perfectly insulated and impermeable though.

This ensemble is as shown in figure:

System 1; Temperature T Volume V Number of Particles N.	**System 2; Temperature T Volume V Number of Particles N.**	**System 3; Temperature T Volume V Number of Particles N.**	**System 4; Temperature T Volume V Number of Particles N.**	**System 5; Temperature T Volume V Number of Particles N.**
System 6; Temperature T Volume V Number of Particles N.	**System 7; Temperature T Volume V Number of Particles N.**	**System 8; Temperature T Volume V Number of Particles N.**	**System 9; Temperature T Volume V Number of Particles N.**	**System 10; Temperature T Volume V Number of Particles N.**
System 11; Temperature T Volume V Number of Particles N.	**System 12; Temperature T Volume V Number of Particles N.**	**System 13; Temperature T Volume V Number of Particles N.**	**System 14; Temperature T Volume V Number of Particles N.**	**System 15; Temperature T Volume V Number of Particles N.**
System 16; Temperature T Volume V Number of Particles N.	**System 17; Temperature T Volume V Number of Particles N.**	**System 18; Temperature T Volume V Number of Particles N.**	**System 19; Temperature T Volume V Number of Particles N.**	**System 20; Temperature T Volume V Number of Particles N.**
System 21; Temperature T Volume V Number of Particles N.	**System 22; Temperature T Volume V Number of Particles N.**	**System 23; Temperature T Volume V Number of Particles N.**	**System 24; Temperature T Volume V Number of Particles N.**	**System 25; Temperature T Volume V Number of Particles N.**

Here, the borders in bold shade are both insulated and impermeable while the borders in light shade are conducting and impermeable.

Grand Canonical Ensemble

It is the collection of a large number of essentially independent systems having the same **temperature T, volume V & chemical potential μ.**

The individual systems of a grand canonical ensemble are separated by rigid permeable and conducting walls. This ensemble is as shown in figure:

System 1; Temperature T Volume V Chemical Potential µ .	**System 2; Temperature T Volume V Chemical Potential µ.**	**System 3; Temperature T Volume V Chemical Potential µ.**	**System 4; Temperature T Volume V Chemical Potential µ.**	**System 5; Temperature T Volume V Chemical Potential µ.**
System 6; Temperature T Volume V Chemical Potential µ.	**System 7; Temperature T Volume V Chemical Potential µ.**	**System 8; Temperature T Volume V Chemical Potential µ.**	**System 9; Temperature T Volume V Chemical Potential µ.**	**System 10; Temperature T Volume V Chemical Potential µ.**
System 11; Temperature T Volume V Chemical Potential µ.	**System 12; Temperature T Volume V Chemical Potential µ.**	**System 13; Temperature T Volume V Chemical Potential µ.**	**System 14; Temperature T Volume V Chemical Potential µ.**	**System 15; Temperature T Volume V Chemical Potential µ.**
System 16; Temperature T Volume V Chemical Potential µ.	**System 17; Temperature T Volume V Chemical Potential µ.**	**System 18; Temperature T Volume V Chemical Potential µ.**	**System 19; Temperature T Volume V Chemical Potential µ.**	**System 20; Temperature T Volume V Chemical Potential µ.**
System 21; Temperature T Volume V Chemical Potential µ.	**System 22; Temperature T Volume V Chemical Potential µ.**	**System 23; Temperature T Volume V Chemical Potential µ.**	**System 24; Temperature T Volume V Chemical Potential µ.**	**System 25; Temperature T Volume V Chemical Potential µ.**

Here inner borders are rigid, permeable and conducting while outer borders are impermeable as well as insulated. As the inner separating walls are conducting and permeable, the exchange of heat energy as well as that of particles between the system takes place, in such a way that all the systems achieve the same common temperature **T** and chemical potential **µ .**

Ensemble Average

Every statistical quantity has not an exact but an approximate value. The average of a statistical quantity during motion is equal to its ensemble average.

If *R(x)* be a statistical quantity along *x*-axis and *N(x)* be the number of phase points in phase space, then **the ensemble average** the statistical quantity *R* is defined as,

$$\bar{R} := \frac{\int_{-\infty}^{\infty} R(x)N(x)\,dx}{\int_{-\infty}^{\infty} N(x)\,dx}$$